MultiDiFlux Evaluation of Selected Multicomponent Couples

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*Multi*DiFlux©

Analysis of Concentration Profiles of Single-Phase Multi-component Diffusion couples for

Interdiffusion Fluxes and Interdiffusion Coefficients

Quantum
Semiconductor
Algorithms

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Selected Multicomponent Systems

- Cu-Ni-Zn System
- Cu-Ni-Zn-Mn Quaternary System
- Rene 88 (GE Couple)
- Fe-Ni-Al

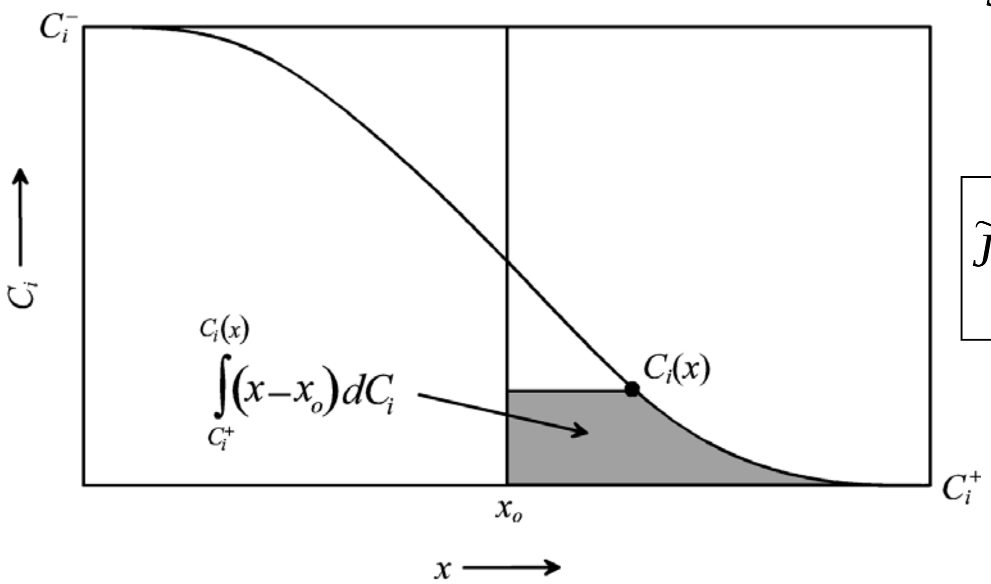
Determination of Interdiffusion Fluxes

from Experimental Concentration Profiles[1,2]

Generalized Fick's Law[3,4]:

$$\tilde{J}_i = -\sum_{j=1}^{n-1} \tilde{D}^n_{ij} \frac{\partial C_j}{\partial x} \quad (i = 1, 2, \ldots, n-1)$$

$$\boxed{\tilde{J}_i(x) = \frac{1}{2t}\int_{C_i^+ \text{ or } C_i^-}^{C_i(x)} (x - x_o)\, dC_i \quad (i = 1, 2, \ldots, n)}$$



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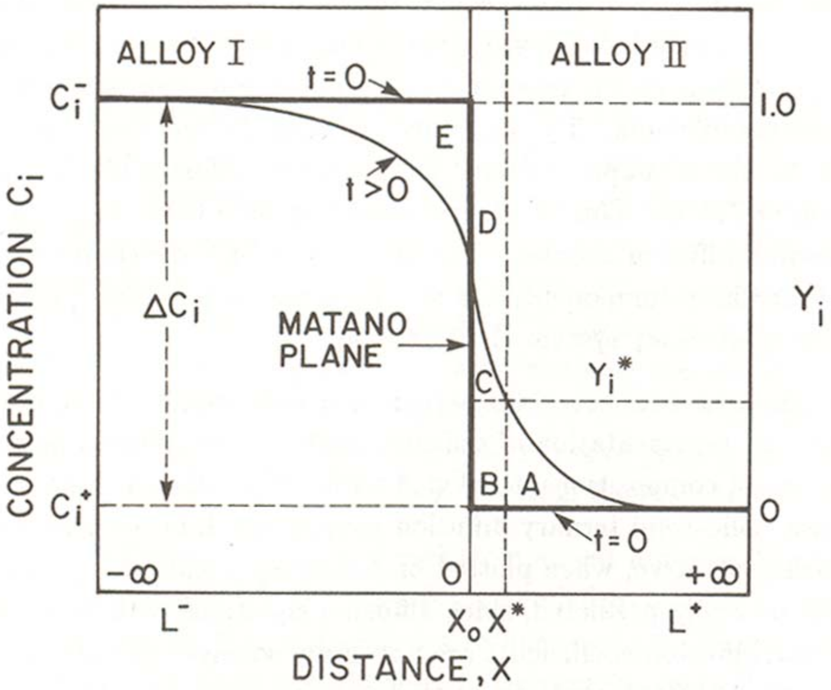
[1] M.A. Dayananda and C.W. Kim, *Metall. Trans. A*, **10A** 1333-1339 (1979).

[2] M.A. Dayananda, *Metall. Trans. A*, **14A** 1851-1858 (1983).

[3] L. Onsager, *Phys. Rev.*, **37** 405-426 (1931).

[4] L. Onsager, *Phys. Rev.*, **38** 2265-2279 (1931).

Flux equations to include molar volume changes



$$Y_i = \frac{C_i - C_i^+}{C_i^- - C_i^+} \qquad (i=1,2,3)$$

$$\mathbf{E+D-A=B+C+D}$$

$$\mathbf{=(x^* - x_o)}$$

$$J_i^M(x^*) = \frac{\Delta C_i}{2t}\left[Y_i^*\int_{-\infty}^{x^*}\frac{(1-Y_i)}{V_m}dx + (1-Y_i^*)\int_{x^*}^{\infty}\frac{Y_i}{V_m}dx\right] \qquad (i=1,2\ldots n)$$

Main Steps carried out by the *Multi*DiFlux Program

- Cubic Hermite polynomial fit to the experimental data for concentration profiles
- Determination of the Matano plane from the profile of each component
- Calculation of interdiffusion fluxes
- Calculation of Interdiffusion coefficients
- Generation of concentration profiles

Basis of *Multi*DiFlux Program

-Integration of Interdiffusion Fluxes-

On the basis of Onsager’s formalism,

$$\tilde{J}_i \qquad \tilde{D}^3_{i1}\frac{C_1}{x} \quad \tilde{D}^3_{i2}\frac{C_2}{x} \qquad (i=1,2)$$

$$\boxed{\begin{aligned} {}^{x_2}_{x_1}\tilde{J}_i dx \qquad & {}^{C_1\ x_2}_{C_1\ x_1}\tilde{D}^3_{i1}dC_1 \qquad {}^{C_2\ x_2}_{C_2\ x_1}\tilde{D}^3_{i2}dC_2 \qquad (i=1,2) \\ & = \overline{\tilde{D}}{}^3_{i1}\ C_1\ x_1 \quad C_1\ x_2 \\ & \quad \overline{\tilde{D}}{}^3_{i2}\ C_2\ x_1 \quad C_2\ x_2 \qquad (i=1,2) \end{aligned}}$$

$\overline{\tilde{D}}{}^3_{i1}$ **and** $\overline{\tilde{D}}{}^3_{i2}$ **are the average values of main and cross interdiffusion coefficients.**

$$\overline{\tilde{D}}{}^3_{ij} = \int_{C_j(x_1)}^{C_j(x_2)} \tilde{D}^3_{ij}dC_j \Big/ \int_{C_j(x_1)}^{C_j(x_2)} dC_j \qquad (i=1,2)$$

$$\tilde{J}_i = -\overline{\tilde{D}}{}^3_{i1}\,\frac{\partial C_1}{\partial x} - \overline{\tilde{D}}{}^3_{i2}\,\frac{\partial C_2}{\partial x} \qquad (i = 1,2)$$

Multiplying by $(x\text{-}x_o)^n$ and integrating over a range of x

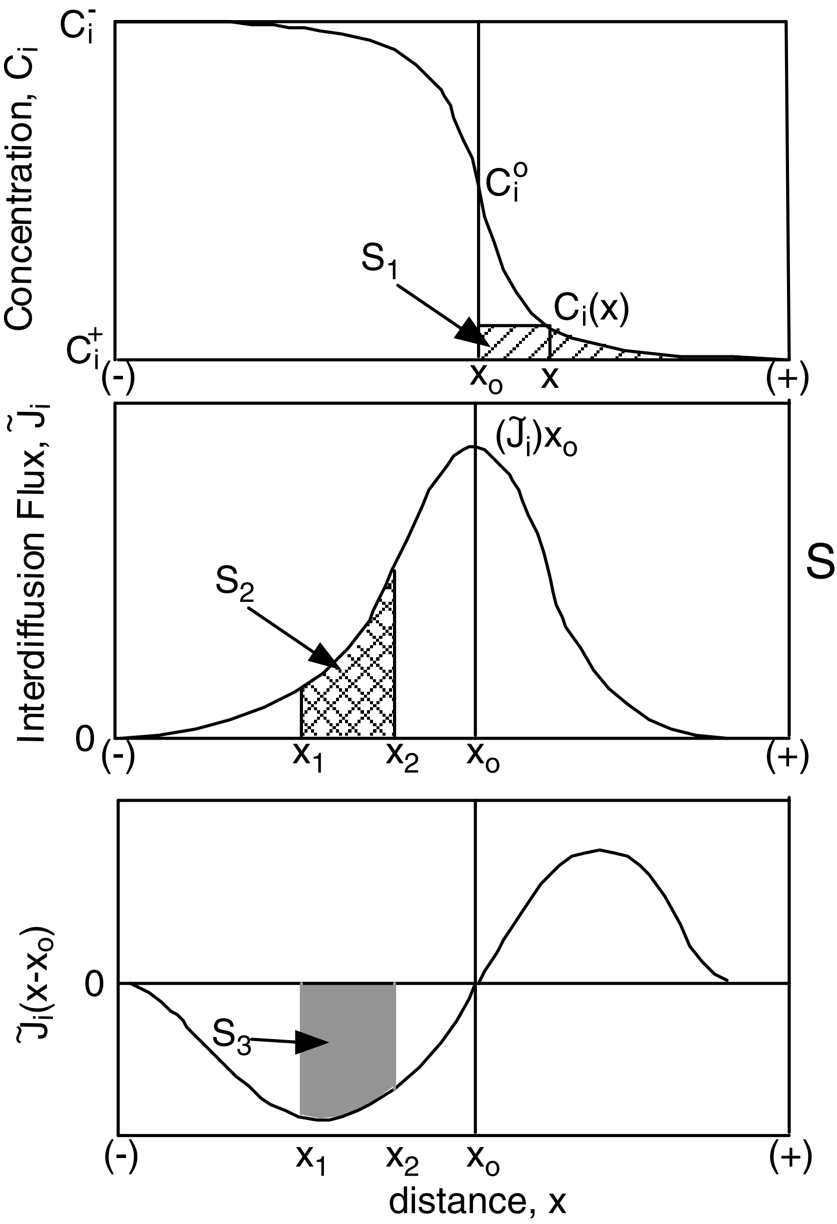
$$\int_{x_1}^{x_2} \tilde{J}_i \left(x - x_o\right)^n dx = -\overline{\tilde{D}}{}^3_{i1} \int_{C_1(x_1)}^{C_1(x_2)} \left(x - x_o\right)^n dC_1 - \overline{\tilde{D}}{}^3_{i2} \int_{C_2(x_1)}^{C_2(x_2)} \left(x - x_o\right)^n dC_2 \qquad (i = 1,2)$$

For n=1

$$\int_{x_1}^{x_2} \tilde{J}_i \left(x - x_o\right) dx = -\overline{\tilde{D}}{}^3_{i1} \int_{C_1(x_1)}^{C_1(x_2)} \left(x - x_o\right) dC_1 - \overline{\tilde{D}}{}^3_{i2} \int_{C_2(x_1)}^{C_2(x_2)} \left(x - x_o\right) dC_2$$

$$= 2t\left\{\overline{\tilde{D}}{}^3_{i1}\left[\tilde{J}_1\left(x_1\right) - \tilde{J}_1\left(x_2\right)\right] + \overline{\tilde{D}}{}^3_{i2}\left[\tilde{J}_2\left(x_1\right) - \tilde{J}_2\left(x_2\right)\right]\right\} \quad (i = 1,2)$$

Determination of $\bar{\tilde{D}}_{ij}^{3}$ $(i, j = 1,2)$ Coefficients



$$S_1 = \int_{C_i^+}^{C_i(x)} (x - x_o)\, dC_i;\quad S_2 = \int_{x_1}^{x_2} \tilde{J}_i\, dx;\quad S_3 = \int_{x_1}^{x_2} \tilde{J}_i (x - x_o)\, dx$$

$$\bar{\tilde{D}}_{ij}^{3} = \int_{C_j(x_1)}^{C_j(x_2)} \tilde{D}_{ij}^{3}\, dC_j \Bigg/ \int_{C_j(x_1)}^{C_j(x_2)} dC_j \qquad (i = 1,2)$$

Regeneration of Concentration Profiles

$$C_1 = K_1\left[\frac{\operatorname{erf}\left(\frac{x-x_0}{2\sqrt{u^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{u^*t}}\right)}{\operatorname{erf}\left(\frac{x_{II}-x_0}{2\sqrt{u^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{u^*t}}\right)}\right]+K_2\left[\frac{\operatorname{erf}\left(\frac{x-x_0}{2\sqrt{v^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{v^*t}}\right)}{\operatorname{erf}\left(\frac{x_{II}-x_0}{2\sqrt{v^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{v^*t}}\right)}\right]+C_{1I}$$

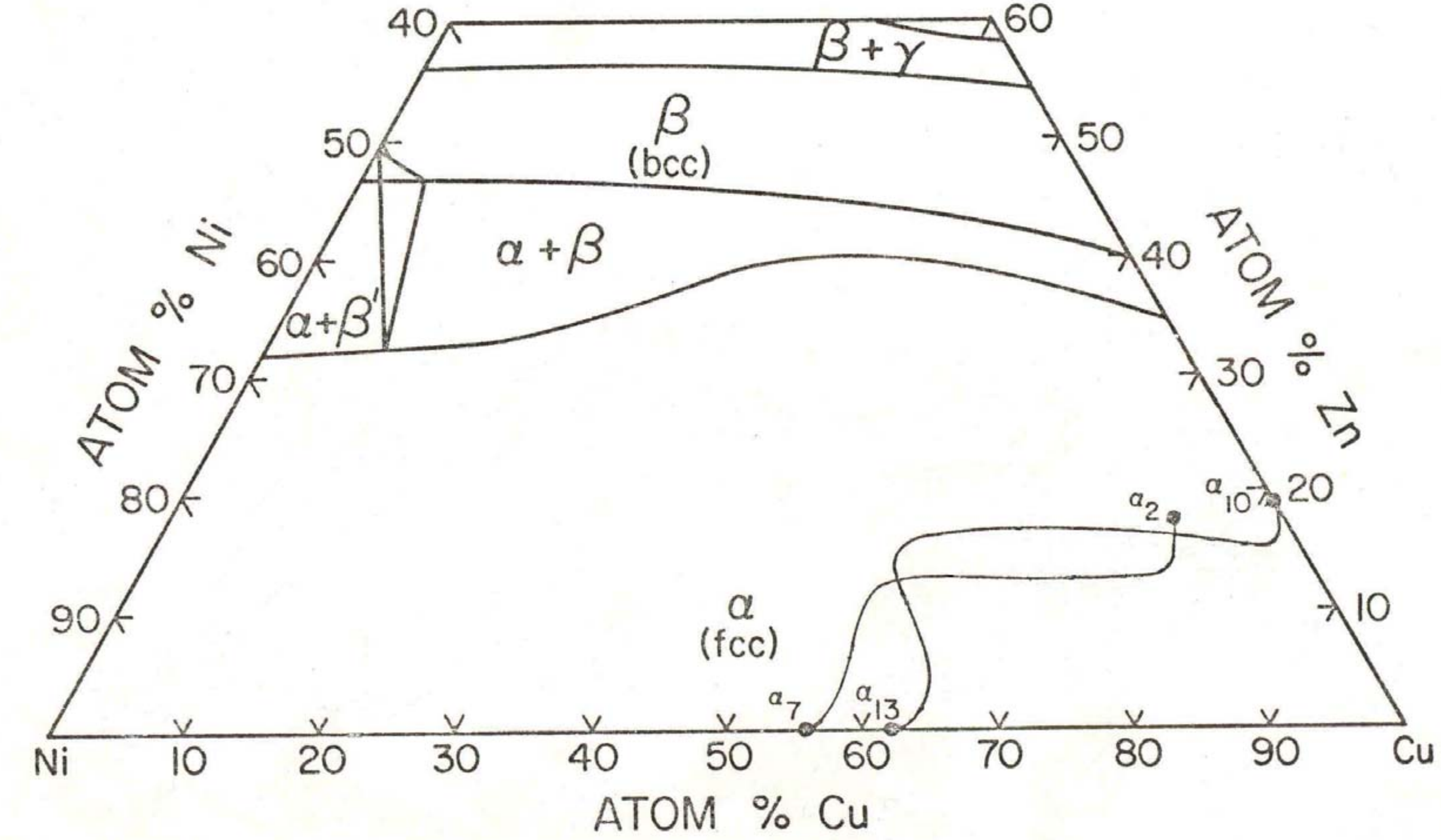
$$C_2 = K_3\left[\frac{\operatorname{erf}\left(\frac{x-x_0}{2\sqrt{u^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{u^*t}}\right)}{\operatorname{erf}\left(\frac{x_{II}-x_0}{2\sqrt{u^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{u^*t}}\right)}\right]+K_4\left[\frac{\operatorname{erf}\left(\frac{x-x_0}{2\sqrt{v^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{v^*t}}\right)}{\operatorname{erf}\left(\frac{x_{II}-x_0}{2\sqrt{v^*t}}\right)-\operatorname{erf}\left(\frac{x_I-x_0}{2\sqrt{v^*t}}\right)}\right]+C_{2I}$$

$$K_1 = \frac{1}{\tilde{D}}\left[\left[\bar{\tilde{D}}^3_{12}\left(C_{2II}-C_{2I}\right)\right]-\left(\bar{\tilde{D}}^3_{22}-\bar{\tilde{D}}^3_{11}-\tilde{D}\right)\left[\frac{C_{1II}-C_{1I}}{2}\right]\right] \quad K_2=\frac{1}{\tilde{D}}\left[\left[\bar{\tilde{D}}^3_{12}\left(C_{2I}-C_{2II}\right)\right]-\left(\bar{\tilde{D}}^3_{22}-\bar{\tilde{D}}^3_{11}+\tilde{D}\right)\left[\frac{C_{1I}-C_{1II}}{2}\right]\right]$$

$$K_3=\frac{1}{\tilde{D}}\left[\left[\bar{\tilde{D}}^3_{21}\left(C_{1II}-C_{1I}\right)\right]-\left(\bar{\tilde{D}}^3_{11}-\bar{\tilde{D}}^3_{22}-\tilde{D}\right)\left[\frac{C_{2II}-C_{2I}}{2}\right]\right] \quad K_4 = \frac{1}{\tilde{D}}\left[\left[\bar{\tilde{D}}^3_{21}\left(C_{1I}-C_{1II}\right)\right]-\left(\bar{\tilde{D}}^3_{11}-\bar{\tilde{D}}^3_{22}+\tilde{D}\right)\left[\frac{C_{2I}-C_{2II}}{2}\right]\right]$$

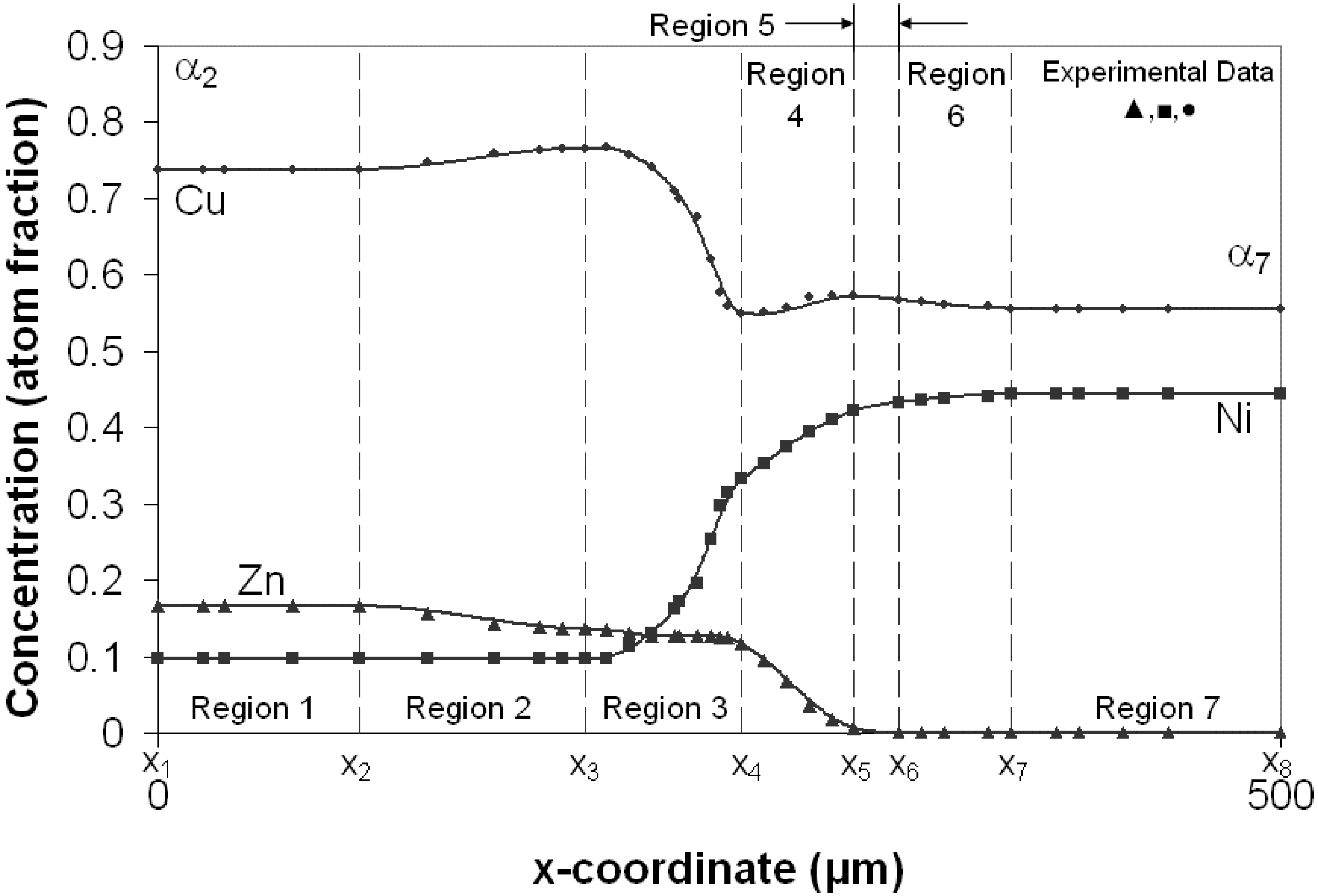
$$u = \bar{\tilde{D}}^3_{11}+0.5\left(\bar{\tilde{D}}^3_{22}-\bar{\tilde{D}}^3_{11}+\tilde{D}\right) \qquad v = \bar{\tilde{D}}^3_{22}+0.5\left(\bar{\tilde{D}}^3_{11}-\bar{\tilde{D}}^3_{22}-\tilde{D}\right)$$

Ternary Cu-N-Zn System at 775ºC
Cu-Isoactivity couples

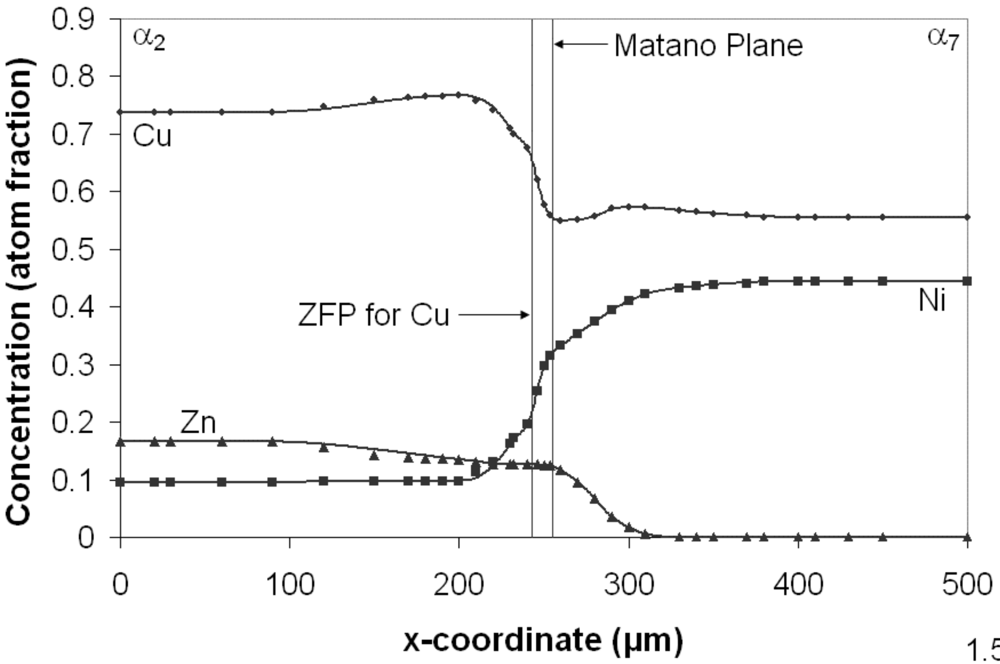


Curve Fitting over Selected Regions of Concentration Profiles

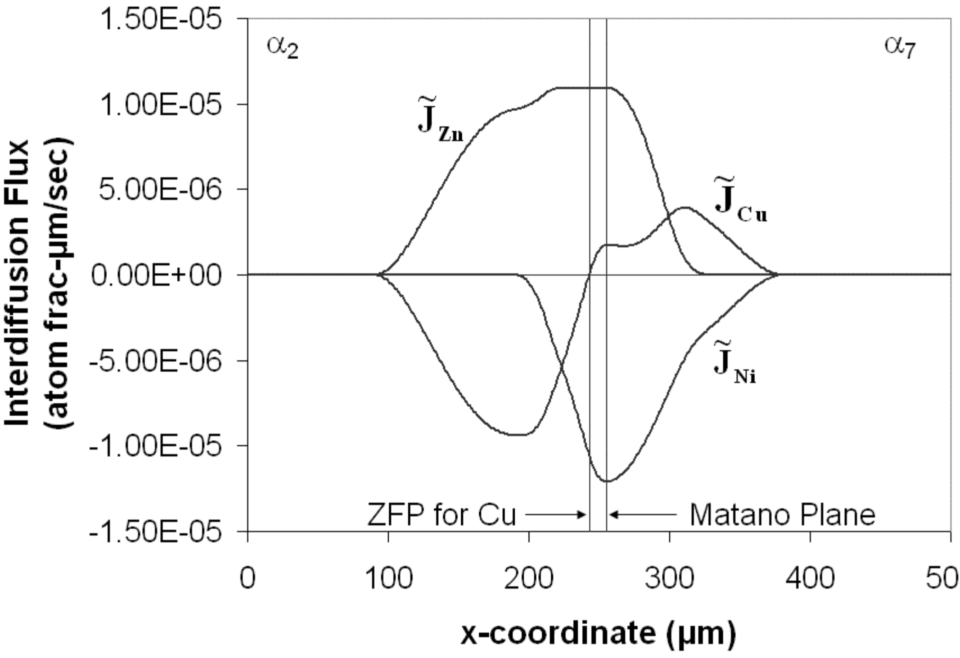
α_2 vs. α_7 couple; 775^{o}C; 2 days



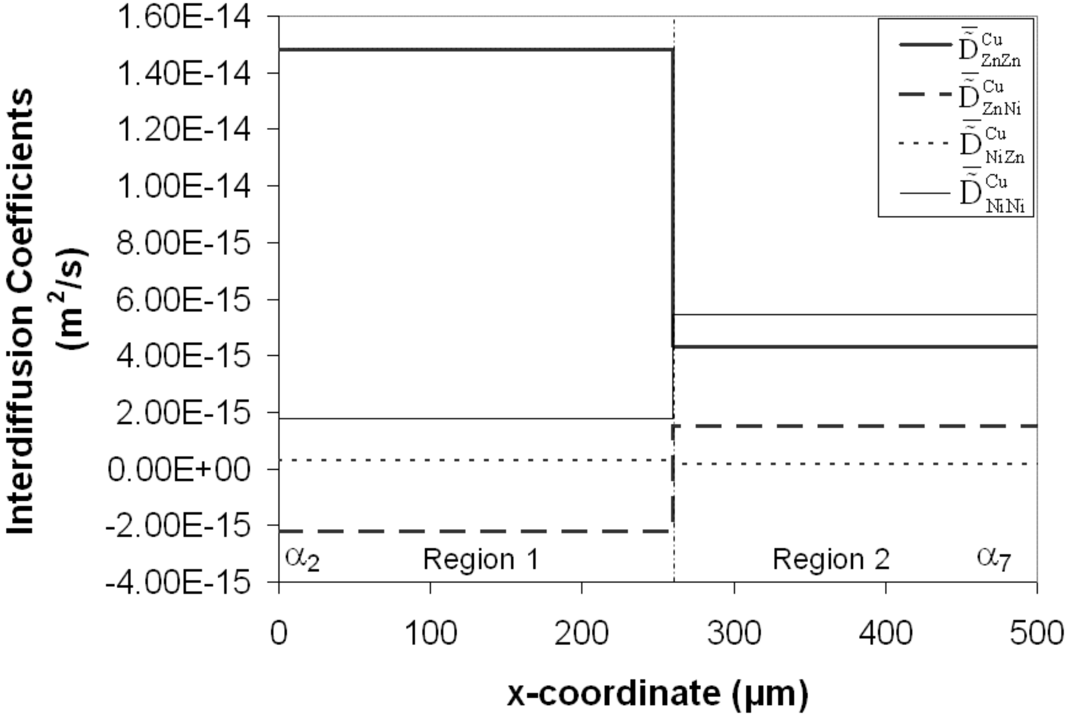
Calculation of Interdiffusion Fluxes



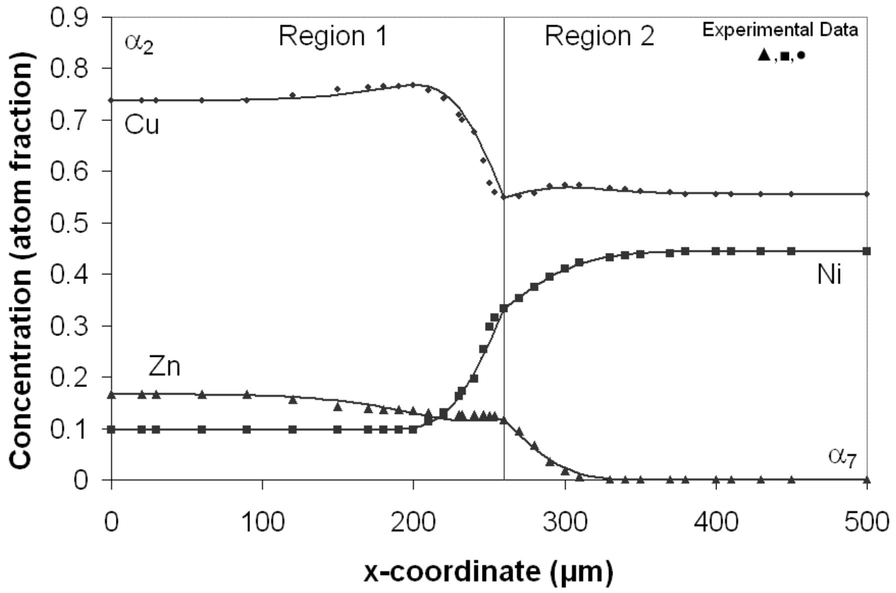
$$\tilde{J}_i = \frac{1}{2t}\int_{C_i^- \text{ or } C_i^+}^{C_i(x)} \left(x - x_o\right) dC_i \qquad \left(i = 1,2,\ldots,n\right)$$



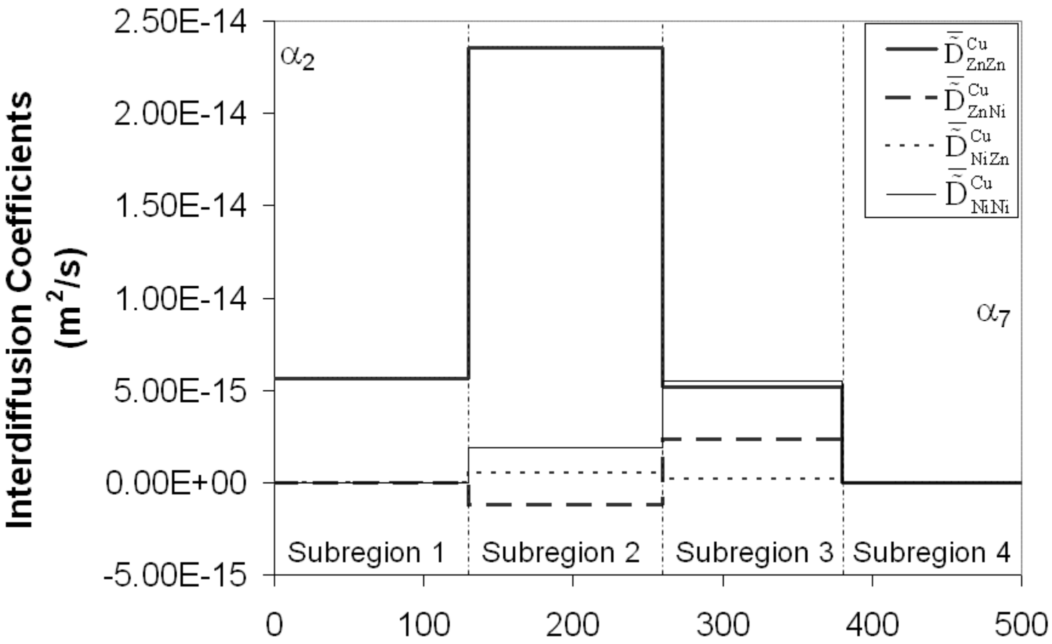
$$\tilde{J}_i(x^*) = \frac{\left(C_i^- - C_i^+\right)}{2t}\left[Y_i^* \int_{-\infty}^{x^*} \frac{\left(1-Y_i\right)}{V_m} dx + \left(1-Y_i^*\right) \int_{x^*}^{+\infty} \frac{Y_i}{V_m} dx\right]$$



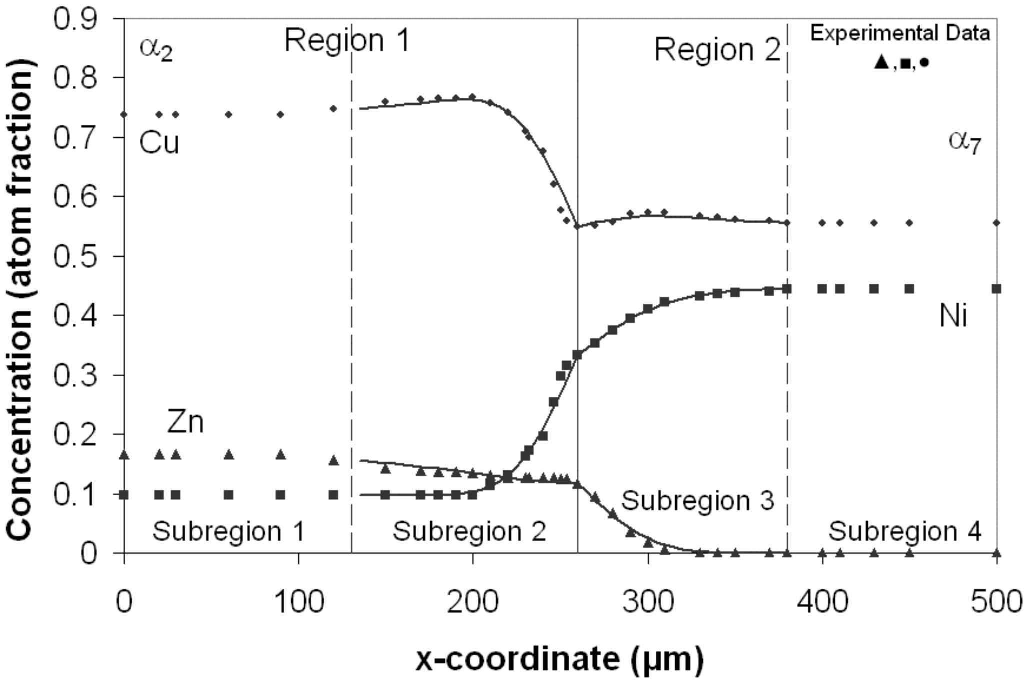
Two sets of calculated interdiffusion coefficients, one on either side of the Matano plane



Regenerated profiles with two sets of interdiffusion coefficient



Four sets of calculated interdiffusion coefficients, two on either side of the Matano plane

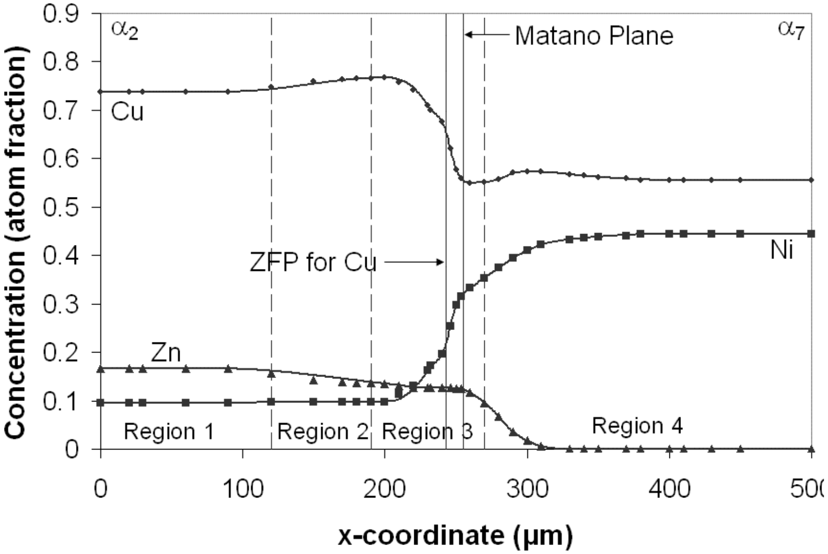


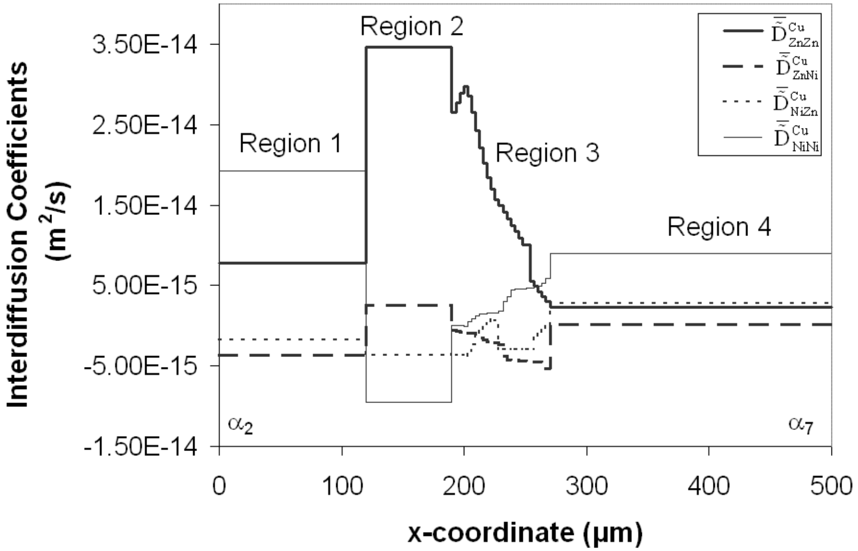
Regenerated profiles with interdiffusion coefficients

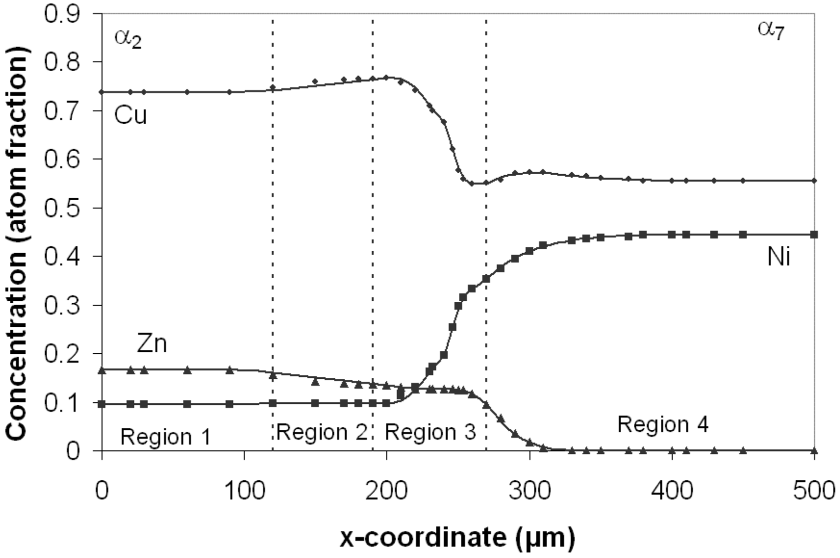
Four sets of interdiffusion coefficients

	Interdiffusion Coefficients			
	$\bar{\tilde{D}}_{ij}^{Cu}\left(\times 10^{-14}\ \mathrm{m}^2/\mathrm{s}\right)$			
x-range	$\bar{\tilde{D}}_{ZnZn}^{Cu}$	$\bar{\tilde{D}}_{ZnNi}^{Cu}$	$\bar{\tilde{D}}_{NiZn}^{Cu}$	$\bar{\tilde{D}}_{NiNi}^{Cu}$
0-130	0.6	0	0	0
130-260	2.4	-0.1	0.05	0.2
260-380	0.5	0.2	0.02	0.6
380-500	0	0	0	0

Reproduction of Concentration Profiles with coefficients over many subregions

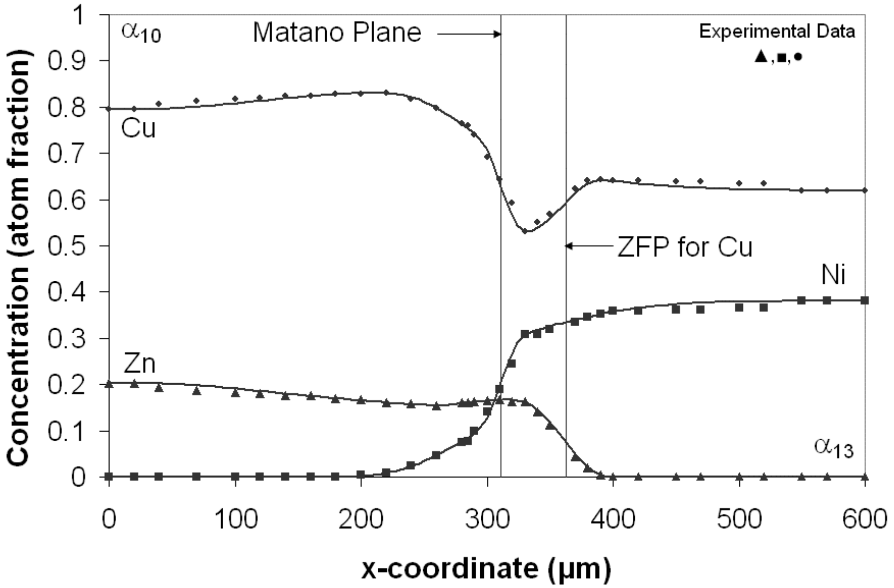


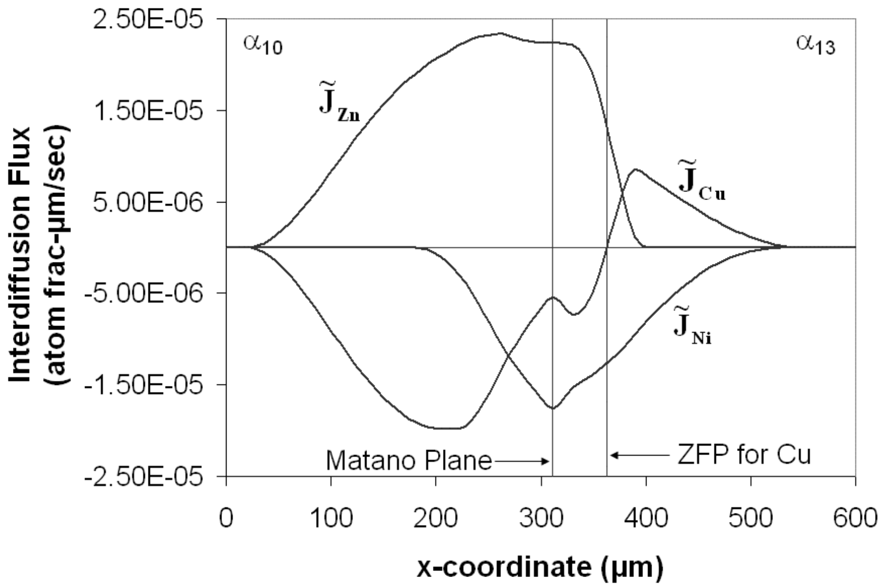




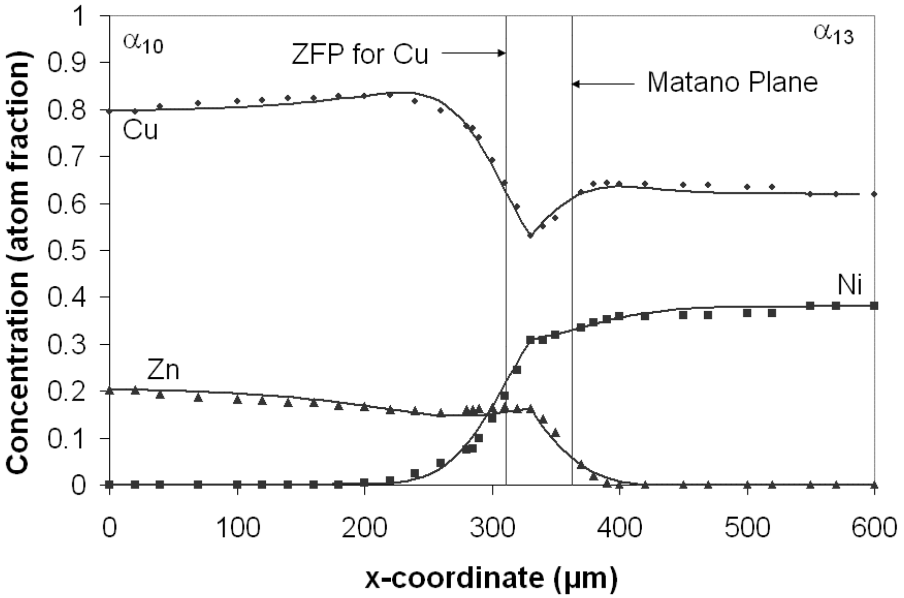
Region 3: 25 Sub-regions

Analysis of a second Cu-isoactivity couple α_{10} vs. α_{13}; 775°C; 2 days





Regeneration of Concentration profiles from two different sets of coefficients



Interdiffusion Coefficients for LHS of Minimum			
$\bar{\tilde{D}}^{Cu}_{ij} \left(\times 10^{-14}\ m^2/s\right)$			
$\bar{\tilde{D}}^{Cu}_{ZnZn}$	$\bar{\tilde{D}}^{Cu}_{ZnNi}$	$\bar{\tilde{D}}^{Cu}_{NiZn}$	$\bar{\tilde{D}}^{Cu}_{NiNi}$
LHS 4.9	-0.9	-0.2	0.4
RHS 0.6	0.2	-0.2	1.3

Interdiffusion Coefficients for RHS of Minimum			
$\bar{\tilde{D}}^{Ni}_{ij} \left(\times 10^{-14}\ m^2/s\right)$			
$\bar{\tilde{D}}^{Ni}_{CuCu}$	$\bar{\tilde{D}}^{Ni}_{CuZn}$	$\bar{\tilde{D}}^{Ni}_{ZnCu}$	$\bar{\tilde{D}}^{Ni}_{ZnZn}$
-0.5	-5.4	0.9	5.8
1.4	1.1	-0.2	0.4

Conversion of Interdiffusion coefficients from one dependent variable to another

$$\tilde{D}^2_{11} = \tilde{D}^3_{11} - \tilde{D}^3_{12}$$

$$\tilde{D}^2_{13} = -\tilde{D}^3_{12}$$

$$\tilde{D}^2_{31} = \tilde{D}^3_{22} + \tilde{D}^3_{12} - \tilde{D}^3_{11} - \tilde{D}^3_{21}$$

$$\tilde{D}^2_{33} = \tilde{D}^3_{22} + \tilde{D}^3_{12}$$

$$\tilde{D}^1_{22} = \tilde{D}^3_{22} - \tilde{D}^3_{21}$$

$$\tilde{D}^1_{23} = -\tilde{D}^3_{21}$$

$$\tilde{D}^1_{32} = \tilde{D}^3_{11} + \tilde{D}^3_{21} - \tilde{D}^3_{22} - \tilde{D}^3_{12}$$

$$\tilde{D}^1_{33} = \tilde{D}^3_{11} + \tilde{D}^3_{21}$$

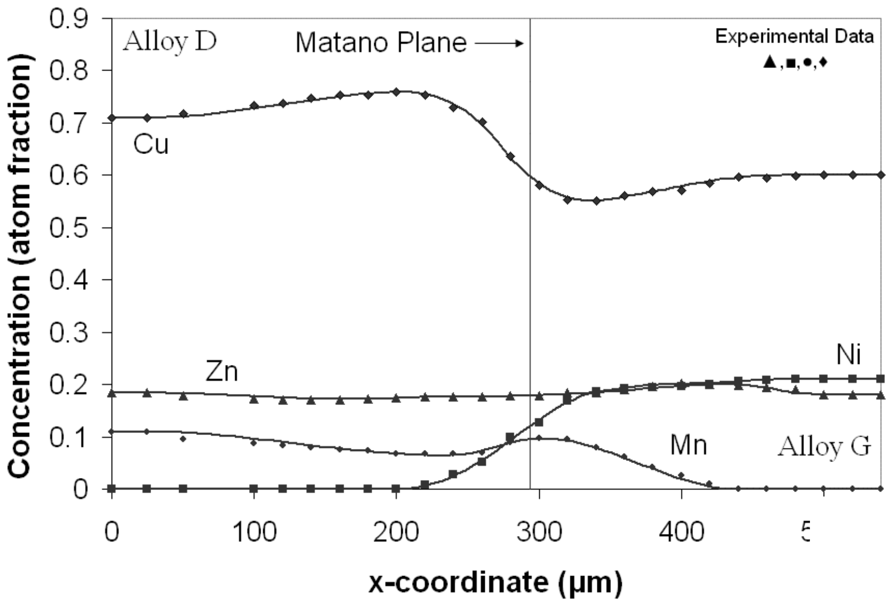
Constraints

$$\tilde{D}^3_{11} + \tilde{D}^3_{22} > 0$$

$$\tilde{D}^3_{11}\tilde{D}^3_{22} - \tilde{D}^3_{12}\tilde{D}^3_{21} \geq 0$$

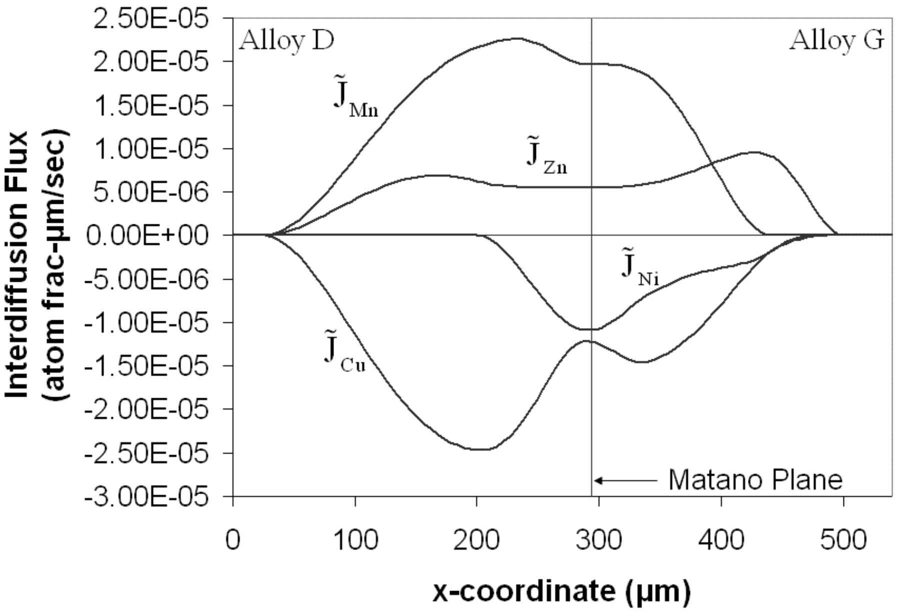
$$\left(\tilde{D}^3_{11} - \tilde{D}^3_{22}\right)^2 + 4\tilde{D}^3_{12}\tilde{D}^3_{21} \geq 0$$

Analysis of a Cu-Ni-Zn-Mn Couple



Alloy D vs. Alloy G couple;

775^{o}C; 2 days



Quaternary Interdiffusion Coefficients for Alloy D vs Alloy G

Interdiffusion Coefficients (Dependent variable is Cu)								
$\bar{\tilde{D}}^{Cu}_{ij}\left(\times 10^{-13}\ m^2/s\right)$								
$\bar{\tilde{D}}^{Cu}_{ZnZn}$	$\bar{\tilde{D}}^{Cu}_{ZnNi}$	$\bar{\tilde{D}}^{Cu}_{ZnMn}$	$\bar{\tilde{D}}^{Cu}_{NiZn}$	$\bar{\tilde{D}}^{Cu}_{NiNi}$	$\bar{\tilde{D}}^{Cu}_{NiMn}$	$\bar{\tilde{D}}^{Cu}_{MnZn}$	$\bar{\tilde{D}}^{Cu}_{MnNi}$	$\bar{\tilde{D}}^{Cu}_{MnMn}$
4.6	-0.3	-1.4	0.6	0.01	-0.2	13.9	-1.0	-4.1
0.1	0.07	0.1	-0.005	0.05	-0.04	-0.6	-0.2	0.03

Interdiffusion Coefficients (Dependent variable is Ni)								
$\bar{\tilde{D}}^{Ni}_{ij}\left(\times 10^{-13}\ m^2/s\right)$								
$\bar{\tilde{D}}^{Ni}_{CuCu}$	$\bar{\tilde{D}}^{Ni}_{CuZn}$	$\bar{\tilde{D}}^{Ni}_{CuMn}$	$\bar{\tilde{D}}^{Ni}_{ZnCu}$	$\bar{\tilde{D}}^{Ni}_{ZnZn}$	$\bar{\tilde{D}}^{Ni}_{ZnMn}$	$\bar{\tilde{D}}^{Ni}_{MnCu}$	$\bar{\tilde{D}}^{Ni}_{MnZn}$	$\bar{\tilde{D}}^{Ni}_{MnMn}$
-0.8	-11.2	2.2	0.3	3.6	-0.7	0.7	8.7	-1.6
-0.8	-0.1	-0.6	-0.2	0.08	0.01	1.4	0.2	0.9

Comparison of ternary to quaternary interdiffusion coeffcients

Interdiffusion Coefficients for α_{10} vs α_{13}

$\bar{\tilde{D}}^{Cu}_{ij}\left(\times 10^{-14}\ m^2/s\right)$

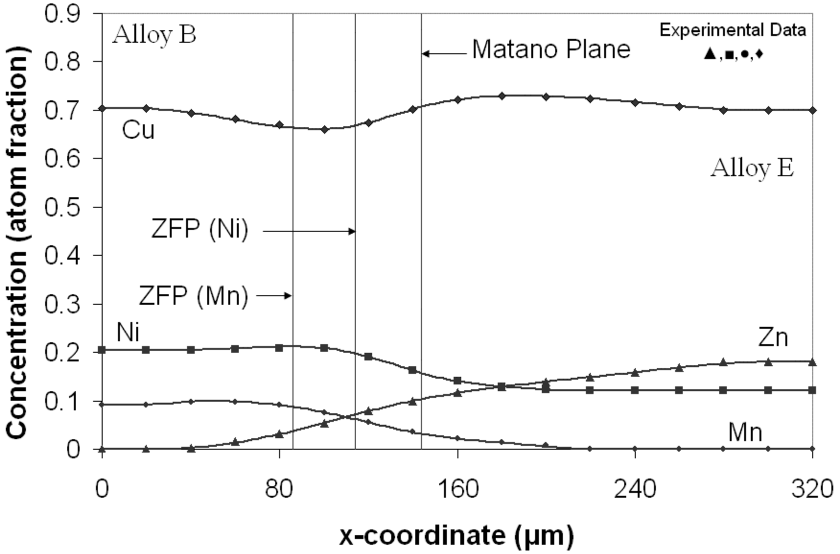
$\bar{\tilde{D}}^{Cu}_{ZnZn}$	$\bar{\tilde{D}}^{Cu}_{ZnNi}$	$\bar{\tilde{D}}^{Cu}_{NiZn}$	$\bar{\tilde{D}}^{Cu}_{NiNi}$
4.9	-0.9	-0.2	0.4
0.6	0.2	-0.2	1.3

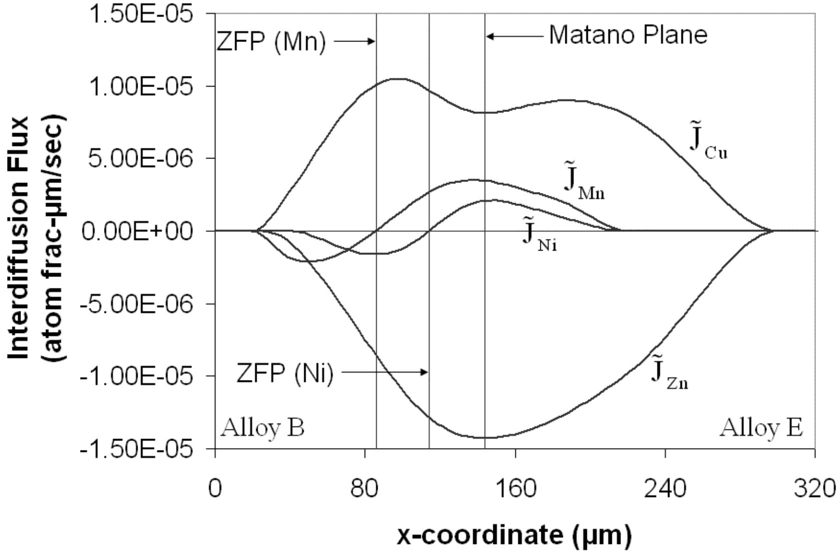
Interdiffusion Coefficients for alloy D vs. alloy G (Dependent variable is Cu)

$\bar{\tilde{D}}^{Cu}_{ij}\left(\times 10^{-13}\ m^2/s\right)$

$\bar{\tilde{D}}^{Cu}_{ZnZn}$	$\bar{\tilde{D}}^{Cu}_{ZnNi}$	$\bar{\tilde{D}}^{Cu}_{ZnMn}$	$\bar{\tilde{D}}^{Cu}_{NiZn}$	$\bar{\tilde{D}}^{Cu}_{NiNi}$	$\bar{\tilde{D}}^{Cu}_{NiMn}$	$\bar{\tilde{D}}^{Cu}_{MnZn}$	$\bar{\tilde{D}}^{Cu}_{MnNi}$	$\bar{\tilde{D}}^{Cu}_{MnMn}$
4.6	-0.3	-1.4	0.6	0.01	-0.2	13.9	-1.0	-4.1
0.1	0.07	0.1	-0.005	0.05	-0.04	-0.6	-0.2	0.03

Alloy B vs. Alloy E Quaternary Cu-Ni-Zn-Mn Diffusion Couple





<table>
<tr><th colspan="9">Interdiffusion Coefficients for alloy B vs. alloy E</th></tr>
<tr><th colspan="9">$\bar{\tilde{D}}^{Cu}_{ij}\left(\times 10^{-14}\ m^2/s\right)$</th></tr>
<tr><th>$\bar{\tilde{D}}^{Cu}_{ZnZn}$</th><th>$\bar{\tilde{D}}^{Cu}_{ZnNi}$</th><th>$\bar{\tilde{D}}^{Cu}_{ZnMn}$</th><th>$\bar{\tilde{D}}^{Cu}_{NiZn}$</th><th>$\bar{\tilde{D}}^{Cu}_{NiNi}$</th><th>$\bar{\tilde{D}}^{Cu}_{NiMn}$</th><th>$\bar{\tilde{D}}^{Cu}_{MnZn}$</th><th>$\bar{\tilde{D}}^{Cu}_{MnNi}$</th><th>$\bar{\tilde{D}}^{Cu}_{MnMn}$</th></tr>
<tr><td>0.6</td><td>-0.4</td><td>-0.2</td><td>0.08</td><td>0.2</td><td>-0.1</td><td>0.2</td><td>0.2</td><td>0.3</td></tr>
<tr><td>0.8</td><td>3.0</td><td>-5.6</td><td>0.006</td><td>0.2</td><td>0.09</td><td>0.01</td><td>0.1</td><td>0.3</td></tr>
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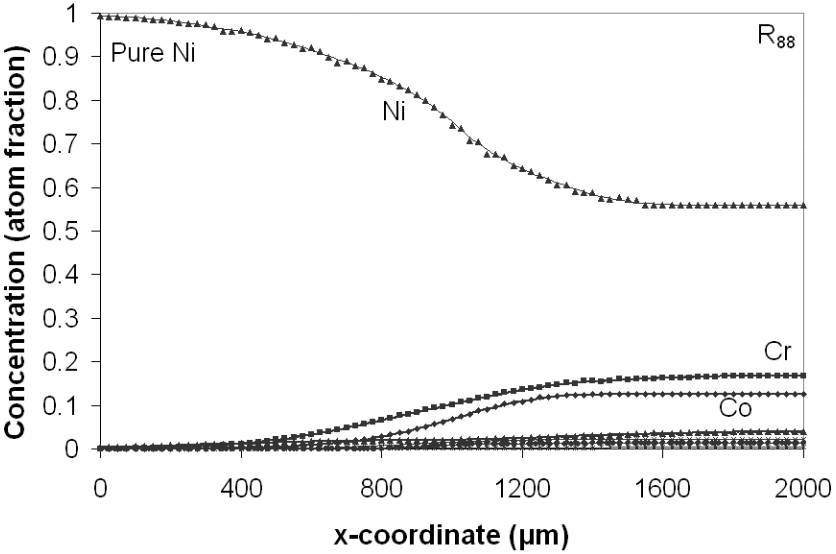
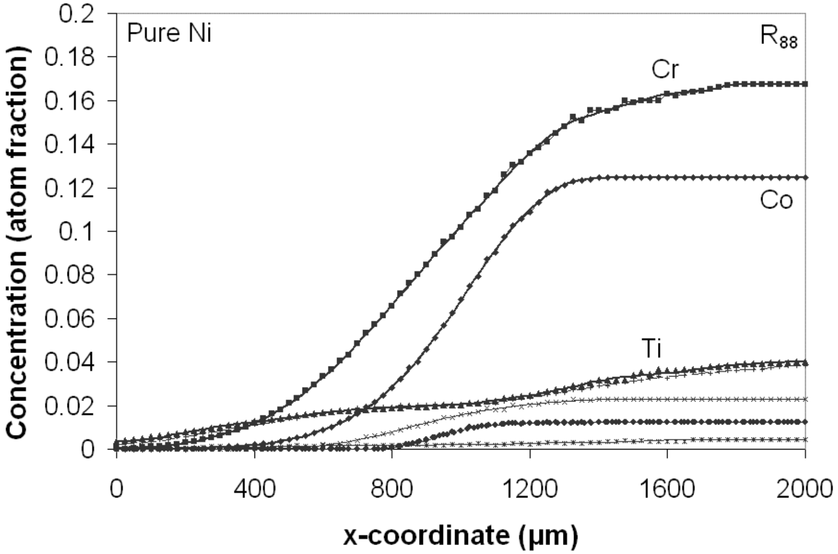
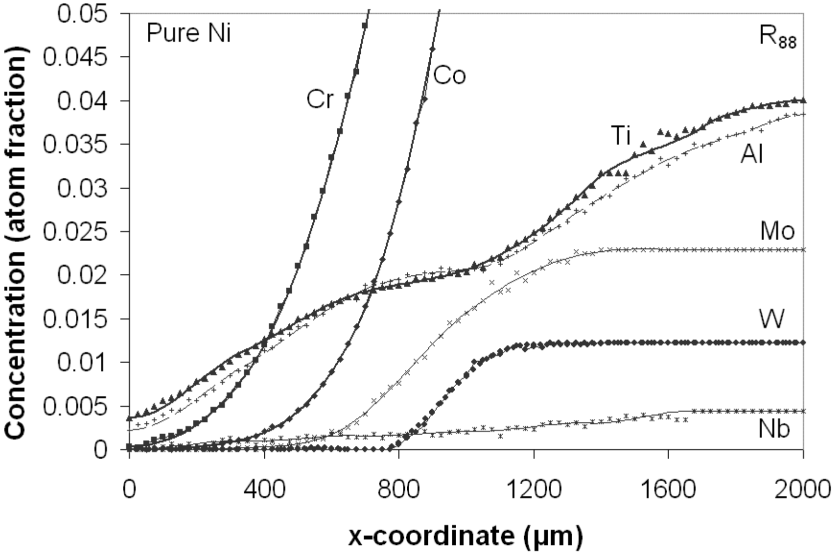
Pure Ni vs R88 ---- 1150oC;1000 hrs (GE)

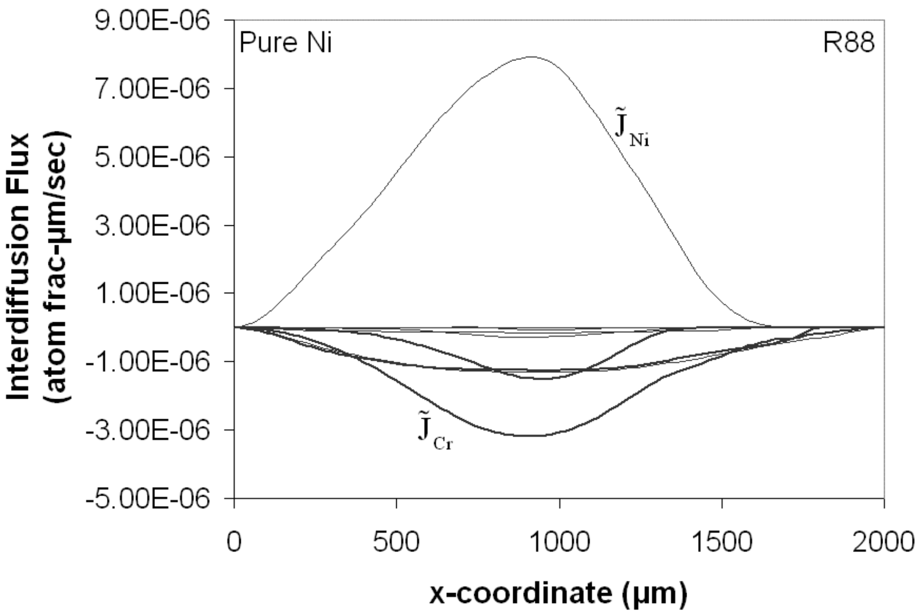


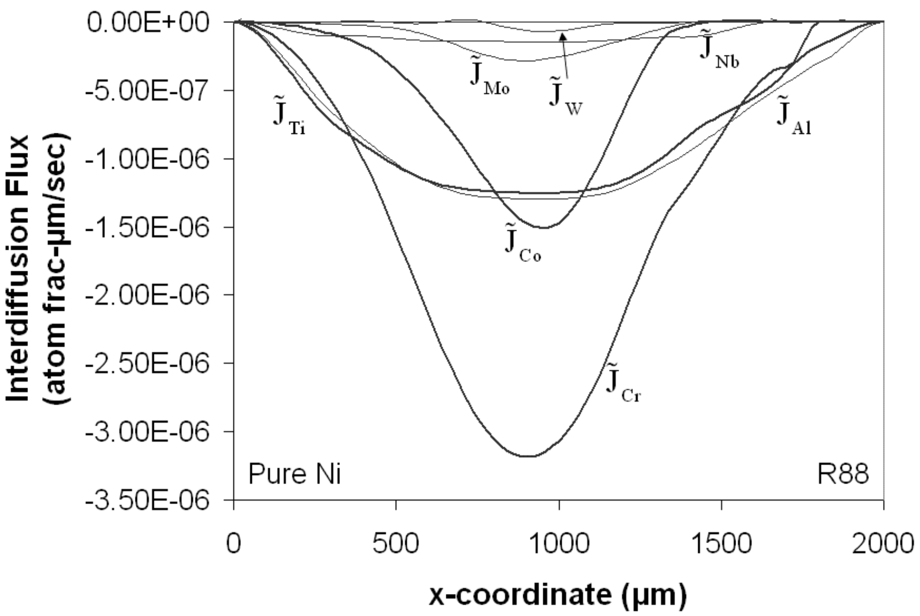




Composition (wt. %) of Rene 88							
Co	**Cr**	**Ti**	**Mo**	**Nb**	**W**	**Al**	**Ni**
13	16	3.7	4	0.7	4	2.1	55.72

Pure Ni vs. R88





Calculation of Effective Interdiffusion Coefficients

$$\tilde{D}^{int}_{i,\Delta C} = \int_{x_1}^{x_2} \tilde{J}_i(x)dx \qquad \tilde{D}^{Int}_{i,\Delta C} = \sum_{j=1}^{n-1} \overline{\tilde{D}^{n}_{ij}}[C_j(x_1) - C_j(x_2)]$$

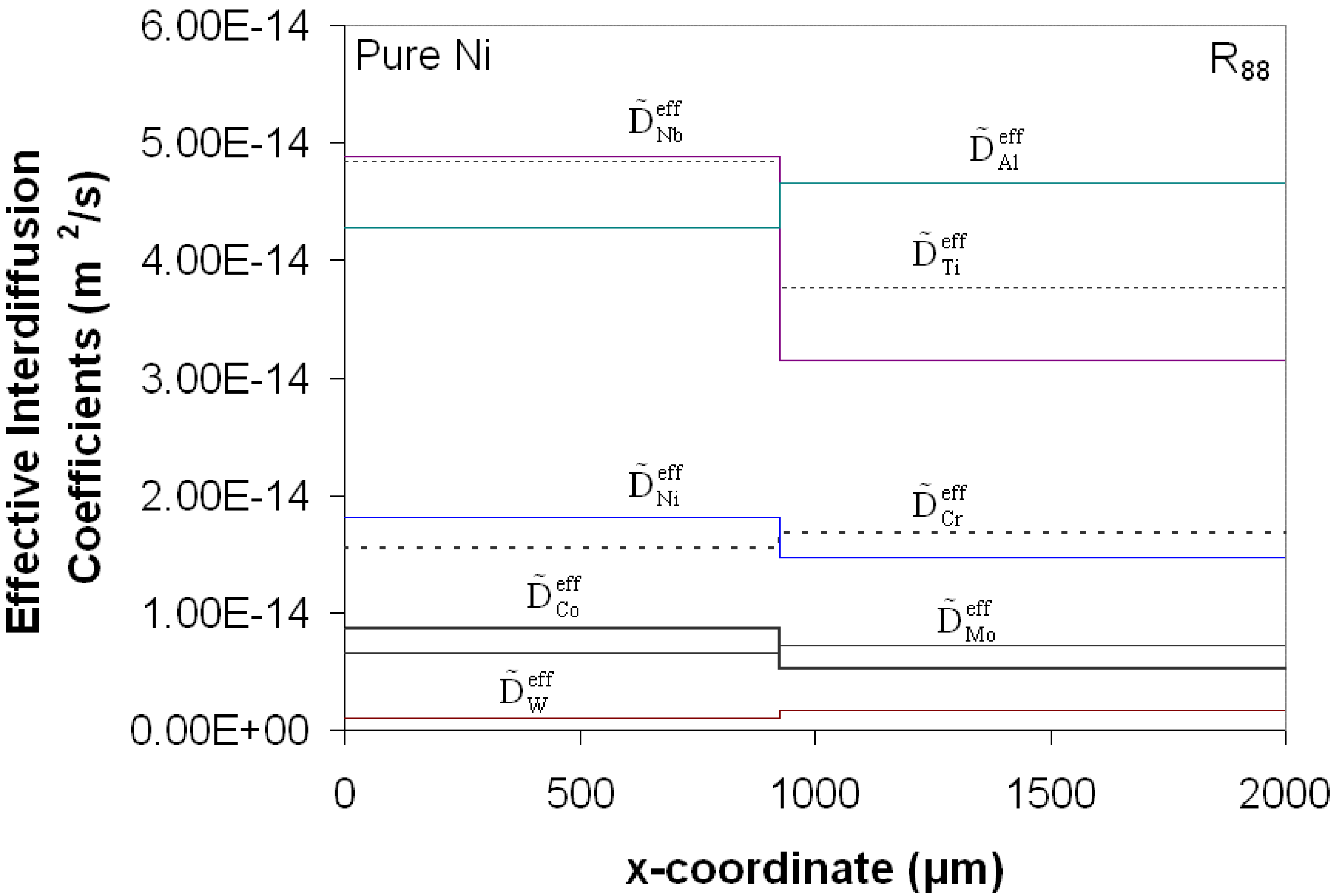
$$\tilde{D}^{eff}_{i,\Delta C} = \frac{\sum\limits_{j=1}^{n-1} \overline{\tilde{D}}^{n}_{ij}\left[C_j(x_1)\text{-}C_j(x_2)\right]}{\left[C_i(x_1)\text{-}C_i(x_2)\right]} \qquad (i=1,2,\ldots,n\text{-}1)$$

- **Dayananda, M.A. Metall. Trans. A,1996, vol. 27A, pp 2504-2509**

Pure Ni vs R88

	Calculated Effective Interdiffusion Coefficients							
	$\tilde{D}^{\mathrm{eff}}_{i,\Delta C}\left(\times 10^{-15}\ \mathrm{m}^2/\mathrm{s}\right)$							
	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Co}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Cr}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Ti}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Mo}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Nb}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{W}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Al}}$	$\tilde{D}^{\mathrm{eff}}_{\mathrm{Ni}}$
$-\infty$ to x_o	8.71	15.5	48.4	6.52	48.9	1.05	42.8	18.1
x_o to $+\infty$	5.21	16.9	37.7	7.19	31.5	1.72	46.5	14.7

Pure Ni vs R88



Equations used to regenerate the profiles from average effective interdiffusion coefficients in an n-component system

$$\tilde{D}_{i,L} = \frac{\int_{C_i^o}^{C_i^-} (x - x_o)^2 \, dC_i}{2t[C_i^- - C_i^o]}$$

$$\tilde{D}_{i,R} = \frac{\int_{C_i^+}^{C_i^o} (x - x_o)^2 \, dC_i}{2t(C_i^o - C_i^+)}$$

$$S = \frac{2\sqrt{\tilde{D}_{i,L} t}\,(C_i^- - C_i^o)}{\alpha_{i,L}}$$

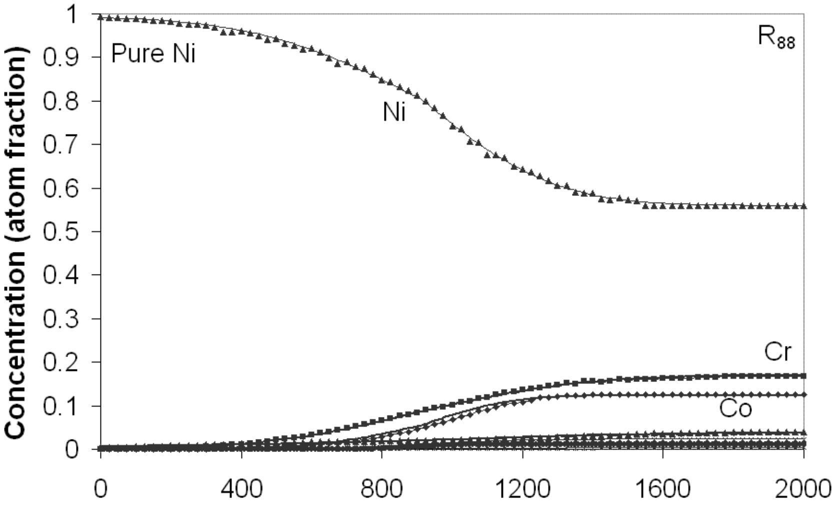
$$S = \frac{2\sqrt{\tilde{D}_{i,R} t}\,(C_i^o - C_i^+)}{\alpha_{i,R}}$$

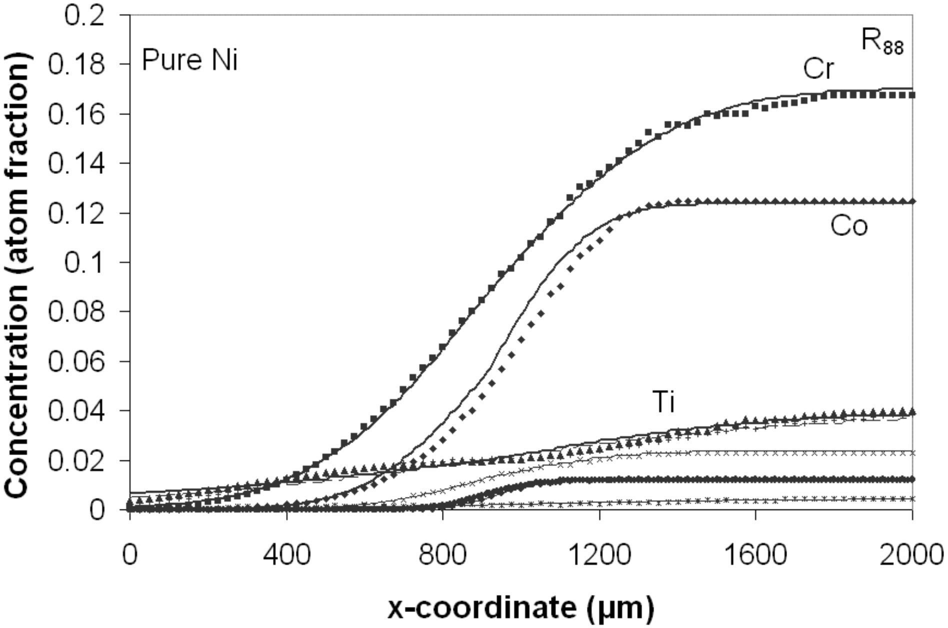
$$C_i = \left[C_i^o - C_i^-\right]\left[1 - erf\left(\frac{\alpha_{i,L}(x_o - x)}{2\sqrt{\pi D_{i,L} t}}\right)\right] + C_i^-$$

$$C_i = (C_i^o - C_i^+)\left[1 - erf\left(\frac{\alpha_{i,R}(x - x_o)}{2\sqrt{\pi D_{i,R} t}}\right)\right] + C_i^+$$

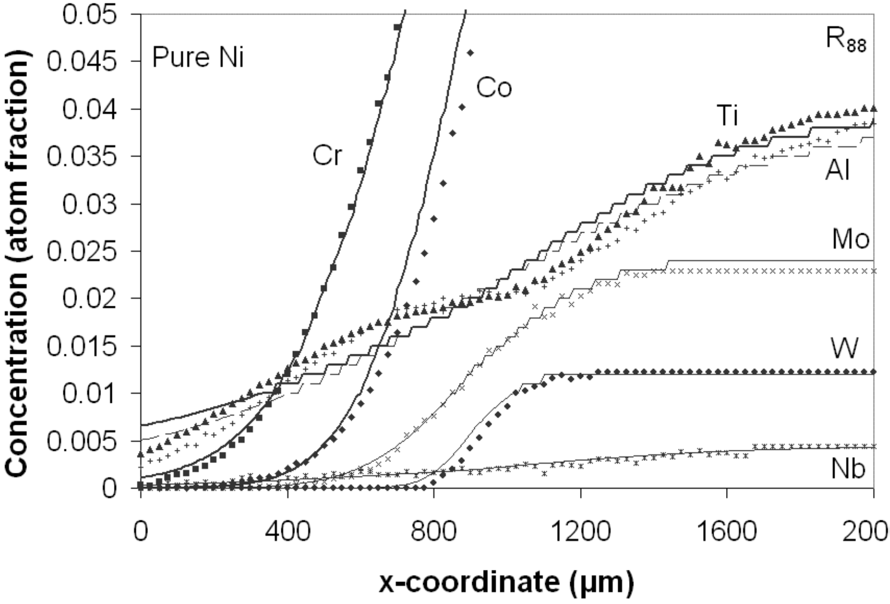
Error Function Representation with Average Effective Interdiffusion Coefficients

Conc Range: 0-1





Conc Range: 0-0.2



Conc Range: 0-0.05

Conclusions

- *The Multi*Diflux program can be used to analyze binary and multicomponent systems (n >2) for interdiffusion fluxes and $(n-1)^2$ interdiffusion coefficients over selected composition ranges.
- All profiles in ternary couples can be analyzed for interdiffusion coefficients and reproduced from the calculated coefficients based on error function solutions.
- The program also calculates an Integrated and an effective interdiffusion coefficient for each component.
- The effective coefficients may also be used to generate concentration profiles in an n-component system.